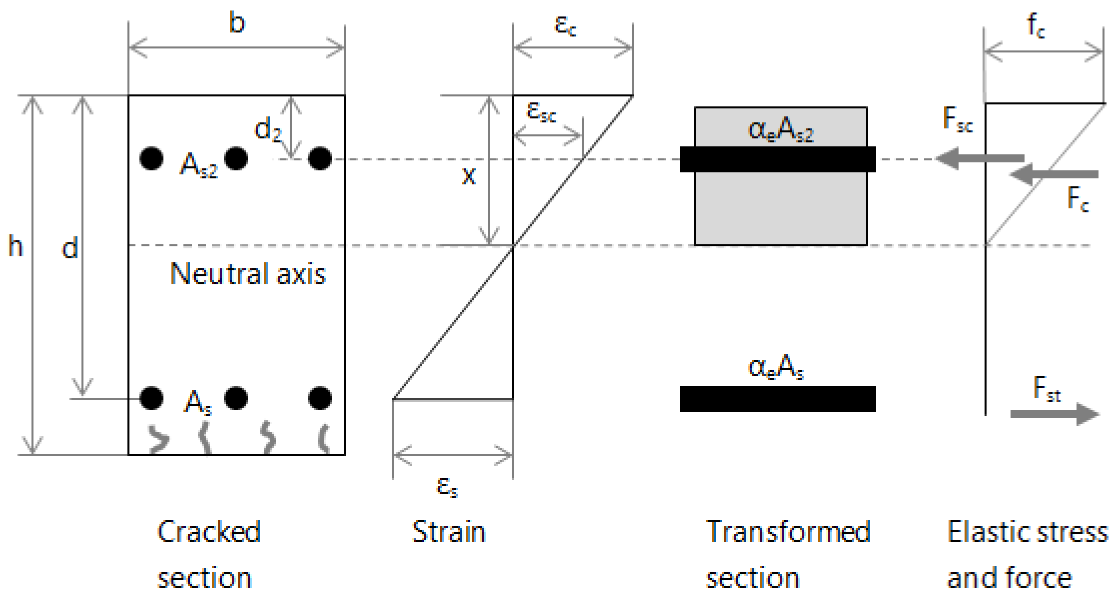
# Crack Width in Doubly Reinforced Concrete Beam

This worksheet calculates the maximum crack spacing and width in doubly reinforced concrete beam, compliant with Eurocode 2




## 1. Properties

### 1.1 Steel

Yield stress $f_y := 420\ \mathbf{MPa}$

Modulus of elasticity $E_s := 2.1 \times 10^5\ \mathbf{MPa}$

### 1.2 Concrete

Unit weight of concrete $\gamma_{concrete} := 25\ \mathbf{kN \cdot m}^{-3}$

Characteristic cylinder strength of concrete $f_{ck} := 32\ \mathbf{MPa}$

Mean value of the tensile strength of concrete
Eurocode 2 Table 3.1 $f_{ctm} := 3.02\ \mathbf{MPa}$

Mean value of the tensile strength of the concrete effective at the time when the cracks may first be expected to occur

$f_{ct\_eff} := f_{ctm}$

Secant modulus of elasticity of concrete
Eurocode 2 Table 3.1

$E_{cm} := 33.3\ \mathbf{GPa}$

Modulus of elasticity of concrete

$E_{c28} := 1.05 \cdot E_{cm}$

Creep coefficient at relevant loading time and duration

$\psi := 1.5$

Effective modulus of concrete

$$E_{eff} := \frac{E_{c28}}{1 + \psi} = 13.986\ \mathbf{GPa}$$

$$\alpha_e := \frac{E_s}{E_{eff}} = 15.015$$

## 1.3 Geometry

Breadth of section

$b := 1000\ \mathbf{mm}$

Overall depth of section

$h := 900\ \mathbf{mm}$

Clear cover of reinforcement

$c_c := 75\ \mathbf{mm}$

Clear cover of compression reinforcement

$c_{c2} := 75\ \mathbf{mm}$

Maximum allow crack width

$w_{limit} := 0.15\ \mathbf{mm}$

Standard rebar diameters

$Dia := [10\ \mathbf{mm}, 12\ \mathbf{mm}, 16\ \mathbf{mm}, 20\ \mathbf{mm}, 25\ \mathbf{mm}, 32\ \mathbf{mm}]$

Cross-section area of rebar

$Area := [seq(\pi \cdot d^2 \cdot 0.25, d\ \mathbf{in}\ Dia)]$

$Area = [78.540\ \mathbf{mm^2}, 113.097\ \mathbf{mm^2}, 201.062\ \mathbf{mm^2}, 314.159\ \mathbf{mm^2}, 490.874\ \mathbf{mm^2}, 804.248\ \mathbf{mm^2}]$

Rebar selection

$Dn := 6$

$s := 150\ \mathbf{mm}$

Diameter of chosen bar

$\phi := Dia[Dn] = 32\ \mathbf{mm}$

Area of reinforcement

$$A_s := Area[Dn] \cdot \frac{b}{s} = 5.362 \times 10^3\ \mathbf{mm^2}$$

$d := h - c_c = 825\ \mathbf{mm}$

$$d_c := c_c + \frac{\phi}{2} = 91\ \text{mm}$$

Rebar selection

$$Dn2 := 5$$

$$s_2 := 200\ \text{mm}$$

Diameter of chosen bar

$$\phi_2 := Dia[Dn2] = 25\ \text{mm}$$

Area of compression reinforcement

$$A_{s2} := Area[Dn2] \cdot \frac{b}{s_2} = 2.454 \times 10^3\ \text{mm}^2$$

$$d_2 := c_{c2} + \frac{\phi_2}{2} = 87.500\ \text{mm}$$

Moment Due to Quasi-Permanent Actions

$$M_{QP} := 400\ \text{kN} \cdot \text{m}$$

## 2. Calculation of Tensile Stress in Reinforcement at Service Loads

### 2.1 Cracking Moment at Moment at Service Loads

Neutral axis depths of uncracked section

$$x_u := \frac{\frac{b \cdot h^2}{2} + (\alpha_e - 1) \cdot (A_s \cdot d + A_{s2} \cdot d_2)}{b \cdot h + (\alpha_e - 1) \cdot (A_s + A_{s2})} = 465.561\ \text{mm}$$

Moment of inertia of uncracked section transformed to concrete

$$I_u := \frac{b \cdot h^3}{12} + b \cdot h \cdot \left(\frac{h}{2} - x_u\right)^2 + (\alpha_e - 1) \cdot \left(A_s \cdot (d - x_u)^2 + A_{s2} \cdot (x_u - d_2)^2\right)$$

$$I_u = 7.559 \times 10^{10}\ \text{mm}^4$$

Cracking moment

$$M_{cr} := f_{ctm} \cdot I_u \cdot \frac{1}{h - x_u} = 525.483\ \text{kN m}$$

### 2.2 Tensile Stress in Reinforcement

Distance from top to neutral axis

$$x := \left( \left( (A_s \cdot \alpha_e + A_{s2} \cdot (\alpha_e - 1))^2 + 2 \cdot b \cdot (A_s \cdot d \cdot \alpha_e + A_{s2} \cdot d_2 \cdot (\alpha_e - 1)) \right)^{1/2} - (A_s \cdot \alpha_e + A_{s2} \cdot (\alpha_e - 1)) \right) \cdot b^{-1}$$

$$x = 275.040\ \text{mm}$$

$$I_{cr} := \frac{b \cdot x^3}{3} + \alpha_e \cdot A_s \cdot (d - x)^2 + (\alpha_e - 1) \cdot A_{s2} \cdot (d_2 - x)^2 = 3.249 \times 10^{10}\ \mathbf{mm^4}$$

Stresses

$$f_{st} := \frac{M_{QP}}{I_{cr}} \cdot \alpha_e \cdot (d - x) = 1.017 \times 10^8\ \mathbf{Pa}$$

$$f_c := \frac{M_{QP}}{I_{cr}} \cdot x = 3.386 \times 10^6\ \mathbf{Pa}$$

$$f_{sc} := \frac{M_{QP}}{I_{cr}} \cdot \alpha_e \cdot (x - d_2) = 3.466 \times 10^7\ \mathbf{Pa}$$

## 2.3 Crack Width (Eurocode 2)

Stress in the tension reinforcement assuming a cracked section — $\sigma_s := f_{st}$

Factor dependent on the duration of the load (0.6 for short term, 0.4 for long term) — $k_t := 0.4$

Depth of effective tension area — $h_{c\_eff} := \min\left(2.5 \cdot (h - d), \frac{h - x}{3}, \frac{h}{2}\right) = 187.500\ \mathbf{mm}$

Effective tension area — $A_{c\_eff} := h_{c\_eff} \cdot b - A_s = 1.821 \times 10^5\ \mathbf{mm^2}$

Effective steel ratio for effective tension — $\rho_{p\_eff} := \frac{A_s}{A_{c\_eff}} = 0.029$

Coefficient of rebar bond property
0.8 for high bond bar
1.6 for prestressing tendons (effectively plain surface
— $k_1 := 0.8$

Coefficient of strain distribution
0.5 for bending, 1.0 for pure tension
— $k_2 := 0.5$

Coefficients found in national annex — $k_3 := 3.4$   $k_4 := 0.425$

Variance in mean strains between reinforcement and concrete.

$$\epsilon_{sm\_cm} := \frac{\sigma_s - k_t \cdot \frac{f_{ct\_eff}}{\rho_{p\_eff}} \cdot \left(1 + \alpha_e \cdot \rho_{p\_eff}\right)}{E_s} = 2.023 \times 10^{-4}$$

$\epsilon_{sm}$ is the mean strain in the reinforcement under the relavent combination of loads

$\epsilon_{cm}$ is the mean strain in the concrete between cracks

$\epsilon_{sm\_cm}$ is $\epsilon_{cm} - \epsilon_{cm}$

Maximum crack spacing

$$s_{r\_max} := k_3 \cdot c_c + \frac{k_1 \cdot k_2 \cdot k_4 \cdot \phi}{\rho_{p\_eff}} = 439.800\ \mathbf{mm}$$

Check

$$check := \begin{cases} \text{"Check Design"} & \epsilon_{sm\_cm} \geq 0.6 \cdot \frac{\sigma_s}{E_s} \\ \text{"OK"} & \text{otherwise} \end{cases}$$

check = "OK"

Crack width
eq 7.8 EN 1992-1-1

$$w_k := s_{r\_max} \cdot \epsilon_{sm\_cm} = 0.089\ \mathbf{mm}$$